台灣口腔醫學科學雜誌
ISSN 1560-1587

牙齒曲拱字符的評估由對傅立葉系列的分析顯現了出從數學上估計的上頷骨齒列

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對於62個上腭齒列，我們參考傳統湯普森牙弓形態學的描述，初步給予分類後，根據參考點、線、角度在先前的主成份分析法中作了探討，並且在向量極座標系統中，說明了特徵向量與前牙的曲度，犬齒前突及牙弓圓滑度，牙弓的長寬比，呈現正比的相關，在現今對於上腭齒列形態特徵的探討中，我們引入了傅立葉分析法研討先前所得的數據，我們發現第二、三、四個傅立葉諧波的振幅與牙弓長寬比，前牙區的曲度，及上腭牙弓的曲線外形，有著極密切的關係。此外，在先前的評估數據和傅立葉分析法中的一系列振幅和常數值間的關係，我們又用了相關分析法的相關系數（P＜0.01）予以檢驗。在現今這個研討的結果，顯示出上腭齒列包含三個要素：牙弓長寬比、前牙區的曲度及牙弓的曲線外形。

關鍵詞：傅立葉分析法、上腭齒列弓

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受文日期：民國97年3月2日
接受刊載：民國97年4月20日

Introduction

For clinical purposes, dental arch forms are expediently classified as square, round-square, round and round V-shaped arches according to Thompson's descriptive morphological study (Thompson & Dewey, 1915). The widely used conventional classification was mainly depending on subjectively visual examination that defines dental arch as upsilon, elliptique, parabolique and hyperbolique forms (Okumura, 1924; Yamazaki, 1934). Some objective methods using mathematical evaluation and quantitative analyses were developed to classify the dental arch forms by translating the measuring data of various morphological elements recorded on plaster casts into algebraical functions (Sakai, 1955; Katayama, 1958). A latter study has also verified hyperbolic-, parabolic- and elliptic-shaped dental arches by a quadratic equation analysis (Biggerstaff, 1972). Another study further superimposed measuring data developed from reference points designated on dental arches with catenary, conic section, elliptic function and fourth polynomial arches, and suggested that the fourth polynomial formula is mostly acceptable for analysing dental arch forms (Iwabayashi, 1977). Although various functions observed by the above methods being useful for mathematical explanations, but seemed difficult to ascertain and compare distinguishing characters of different dental arch types.

Morphological study by using Fourier analysis has been proposed being effective for statistical evaluation of the mandibular fossa (Ohba *et al.,* 1976; Kashiwagi, 1977), tooth (Ozaki *et al.,* 1977), inferior border of mandible (Soma, 1980), the dental arch (Sekimoto, 1983; Sekikawa, 1986; Tsuji *et al.,* 1986; Sekimoto *et al.,* 1988; Kasai *et al.,* 1997; Lestrel *et al.,* 2004) and positional relationships between the palatine and dental arch. In the present study, we try to evaluate dental arch forms with Fourier analysis, because 1) it can get hold of, estimate and compare the numerical information representing sizes and forms of dental arches, and 2) by intersection of Fourier arithmetical (geometrical) series with certain harmonics, coefficients are reproducible for further statistical analysis. The present study aims at the dissolution of relationships of essential factors that affect maxillary dental arch forms by using Fourier analysis on estimated items developed on maxillary dentitions in our previous study (Nakatsuka *et al.,* 2004).

Materials and Methods

1. Materials

The Taiwan J Oral Med Sci 2008; 24:86-104
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The study was performed following the Declaration of Helsinki Ethical Principles for Medical Research (1964). The protocol (No. 05507: Morphological Analysis of dental arches) was approved by the Ethic Review Board of Osaka Dental University. In the study, 396 sets of paired maxillary and mandibular dental casts of young adult students (18 to 26 years old; male: 257, female: 139) at Osaka Dental University were prepared to have a total height of 60 mm with the occlusal plane adjusted to be at a height of 30 mm from the mandibular cast base (Nakatsuka et al., 2004). We selected 62 sets (male: 36; female: 26) with normal dentition and occlusion, and the maxillary dentitions of the standardised specimens were preliminarily classified as square, round-square, round and round V-shaped arches.

2. Methods

Designation of references and lines for examination

We designated the midpoints of the incisor edge ($I1_R$, $I1_L$, $I2_R$ & $I2_L$), the summits of the cuspids (C_R & C_L), the buccal cusps of premolars ($P1_R$, $P1_L$, $P2_R$ & $P2_L$) and the mesial buccal cusps of the first and second molars ($M1_R$, $M1_L$, $M2_R$ & $M2_L$) to be the main reference points. The midpoint of line ($I1_R$-$I1_L$) was defined a reference point A. From A, let a vertical line intersected line ($M2_R$-$M2_L$) at reference point B. The line (A-B) intersected line (C_R-C_L) at reference point E. Point O (origin, pole) was the reference midpoint at the line ($M2_R$-$M2_L$) (original line), and was designated to study the relationships between right and left radius vectors $r\theta n$ (n=1, ⋯ 7) $_{R\&L}$ (θ=angles between line (O-reference points) and the original line ($M2_R$-$M2_L$)) by developing a spherical (polar) coordinate system (Sekikawa, 1986; Nakatsuka *et al.,* 2004; Fig. 1).

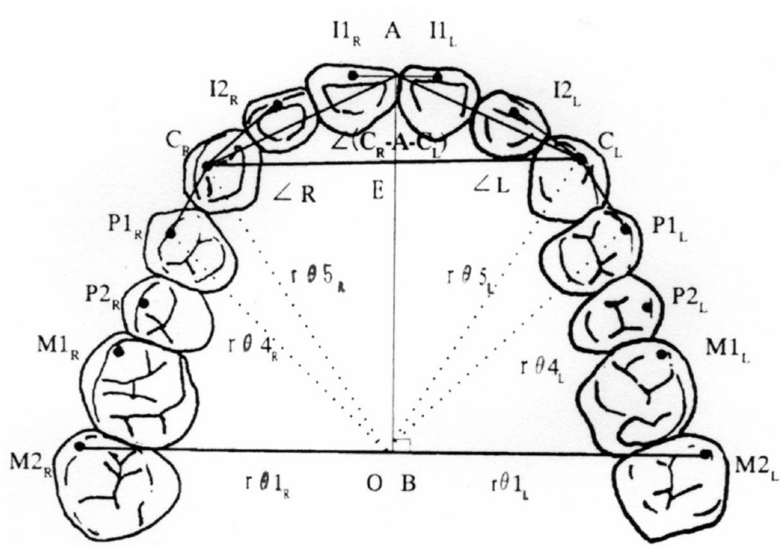

Fig.1：Reference points, lines, and angles. A : the midpoint of line (I1R-I1L), B: let a vertical line intersected line (M2R-M2L) from A, E: the line (A-B) intersected line (CR-CL), O(origin, pole): the midpoint at the line (M2R-M2L) (original line), $r\theta n$ (n=1, ⋯7) R&L (θ=angles between line (O-reference points) and the original line (M2R-M2L))

Photography of the dental casts

A Nikon AD100 digital camera fitted with an AF MICRO Nikkor 70-180 mm, 1:4.5-5.6 D lens (Nikon, Tokyo, Japan) was set up on a King L4 Copy Stand (Asanuma, Tokyo, Japan). The axis of the lens was aligned vertically to the occulsal plane (standard plane) of mandibular casts (focusing distance=70 mm, f=29), and the distance between the lens and standard plane was adjusted to 310 mm so that the photographs keeping in equal proportion with the casts. The digital data were processed with Adobe Photoshop CS (Adobe System; Cal., USA) and Adobe Page Maker (ver. 6.5J, Adobe System) on a Dell Optiplex GX200 computer (OS: Microsoft Windows XP professional; DELL Japan, Kanagawa, Japan), and the equal-proportional images of plaster casts were printed out by using an Epson LP-8300C colour printer (Epson, Nagano, Japan).

Establishment and development of reference items

By connecting reference points printed on photos, we established 6 items for differentiating maxillary dental arches: ① $\angle R + \angle L$, ② $(A-B)/(C_R-C_L)$, ③ $180° - \angle(C_R\text{-}A\text{-}C_L)$, ④ $(A-E)/(C_R-C_L)$, ⑤ $(A-B)/(M2_R-M2_L)$ and ⑥ $(r\theta 5-r\theta 4)_R + (r\theta 5-r\theta 4)_L$ (Fig. 1). The definition of items has been described in our previous study (Nakatsuka *et al.,* 2004).

Fourier series

The Fourier series is a system that its optional periodic functions of 2π periodicity can be represented by the composition of trigonometric functions with different periodicity and amplitude (Kawada, 1985; Ohishi, 1989; Funakoshi, 1997) . The present study was evaluated by Origin (R) 6.1J (Origin Lab Co, MA, USA) with the following formula:

$Y(\theta i) = a_0 + \Sigma(a_i{*}\cos\theta i + b_i{*}\sin\theta i)$ (i= 1, 2,···,7)

$2a_0$: constant value; representing the dental arch size (Sekimoto, 1983; Sekikawa, 1986; Tsuji *et al.,* 1986; Sekimoto *et al.,* 1988; Kasai, 1997).

a_i , b_i: Fourier coefficients.

$\sqrt{a_i^2+b_i^2}$: the associated wave of cosine (cos) and sine (sin) waves with a same harmonic: amplitude; representing the dental arch form.

The $2a_o$ is also an area obtained by dividing the $Y(\theta i)$ graph with a periodicity T (Sekimoto, 1983; Kawata, 1985; Sekikawa, 1986; Tsuji *et al.,* 1986; Sekimoto *et al.,* 1988; Ohishi, 1989; Kasai *et al.,* 1997; Funakoshi, 1997). On the other hand, a_i and b_i are Fourier coefficients of cos

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and sin waves, respectively, and $\sqrt{ai^2+bi^2}$ is developed to indicate the amplitude obtained by combining the cos and sin waves of a same harmonic. It also expresses the size implicating the combined wave, that we can evaluate how the weight (contribution) affecting the wave. Standardization of the constant value 2 a_o has been proposed for correction of individual differences, while it increases the risk of error during processing the finite term in a Fourier series; in the present study, we did not standardize the constant value 2 a_0 (Soma, 1980; Sekimoto, 1983; Sekikawa, 1986).

We defined 14 reference points on a maxillary cast and let the wave went up-and-down less than 7 times in a period (Sekikawa, 1986). The present Fourier analysis defined and investigated i= 1, 2,⋯,7 (i= 1 represented the wave of the 1st harmonic and so on). The morphological characters of the constant value 2 a_o and amplitudes $\sqrt{ai^2+bi^2}$ of the associated waves were estimated as followings:

1) Comparison of the contribution and cumulative contribution expressed in different arch forms by studying amplitude of different harmonics.
2) Comparison of the constant value and the amplitude of different arch forms.

   One-factor ANOVA ($p<0.01$) and Kruskal Wallis test ($p<0.01$) for non-homogenous dispersive data were processed using Excel Statistics 2000 for Windows (Community Information Services, Tokyo, Japan). Significant data were assessed by Scheffe’s test for *post hoc*.
3) Comparison of measured data with the constant value and amplitude.

   One-factor ANOVA ($p<0.01$) and Kruskal Wallis test ($p<0.01$) for non-homogenous dispersive data were processed using Excel Statistics 2000 for Windows (Community Information Services, Tokyo, Japan). Significant data were assessed by Scheffe’s test for *post hoc*. Furthermore, correlation coefficient analysis ($p<0.01$) on the relationships between the present Fourier analysis and the results obtained in our previous study was also performed (Aoki, 1989; Endo & Yamamoto, 1992; Suga, 1993; Uchida, 1997; Agata, 1997; Katahira, 1999; Okuda, 1999; Nakatsuka *et al.,* 2004).

In this study, we in particular estimated the recurrence in percentage (recurrence %) by superimposing the amplitude of different harmonics with the summated amplitudes $\sqrt{ai^2+bi^2}$ (from 1st to 7th harmonics), and defined the c0ontribution for each dental arch type. The cumulative contribution was the contribution developed from accumulation of the amplitudes from 1st to 7th harmonics, and thus reproduced arch type forms with high recurrence not less than 90% (Sekikawa, 1986).

Results

1. Contribution and cumulative contribution of dental arch forms

All of the four types (square, round-square, round and round V-shaped arches) showed approximately 90% of cumulative contribution; we observed higher cumulative contribution percentage in 1st to 3rd harmonics, and the 2nd harmonic had the highest contribution in all arch forms (Table 1, Fig. 2). The 4th harmonics of square and round arches, 5th harmonic of round arches and 3rd harmonic of round V-shaped arches had cumulative contribution nearly at 90% high.

2. Average morphology of each dental arch form

We calculated constant values and the mean of Fourier coefficients of 1st- to 4th-harmonic for drawing average curves of maxillary arches (solid line: mean of

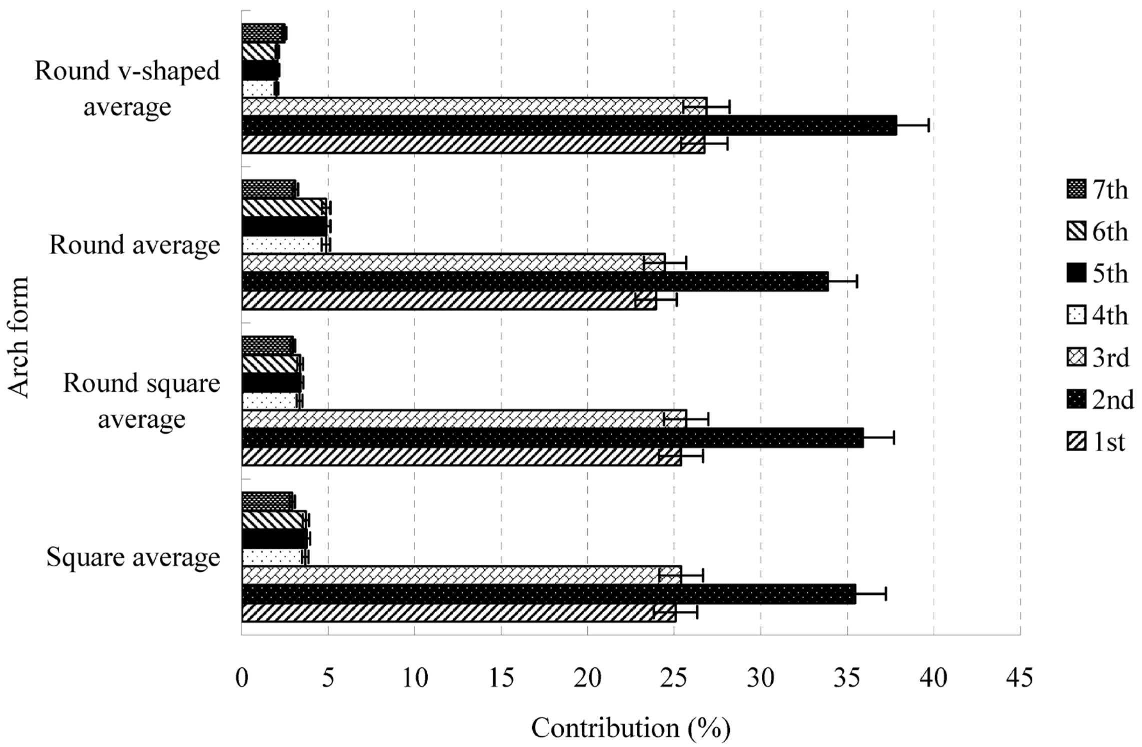

Fig. 2：Contributions of the harmonics developed from the average of each arch type.

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original data, dotted line: 1^{st} harmonic, broken line: $1^{st}+2^{nd}$ harmonics, 1-dot chained line: $1^{st}+2^{nd}+3^{rd}$ harmonics, 2-dot chained line: $1^{st}+2^{nd}+3^{rd}+4^{th}$ harmonics; Figs. 3-6). The 1^{st} harmonic curves of all arch forms rather deviated from original data, while the $1^{st}+2^{nd}$-harmonic curves overlapped with the original data in particular at the molar segments; the findings indicated that there were certain characters of each arch form. We observed that the curves for $1^{st}+2^{nd}+3^{rd}$-harmonic waves increased in adaptation at the anterior teeth region, while $1^{st}+2^{nd}+3^{rd}+4^{th}$-harmonic curves reproduced different arch forms with the original data. In accordance with a previous study, we also observed that the mid-portion usually adapted more than the ends of combined-harmonic curves with original-data curves (Tsuji, 1986). The results suggested that the $1^{st}+2^{nd}+3^{rd}$- and $1^{st}+2^{nd}+3^{rd}+4^{th}$-harmonic curves were nearly completely fitted with the original data of all arch forms.

3. Superimposition of average dental arch forms

We superimposed the average forms of maxillary dental arches (the $1^{st}+2^{nd}+3^{rd}+4^{th}$ curve of each arch form; broken line: square arch, solid line: round-square, 2-dot chained line: round arch, dotted line: round V-shaped arch), and found that square arches had a distinct projection of the cuspid regions and round V-shaped arches had a pointing anterior-teeth region. Two endings (molar regions) of round arches showed an increase of values, but the other arch types showed a decrease followed by an increase of the values at the molar regions.

Table 1：Contributions and cumulative contributions of harmonics developed from means amplitudes of different dental arch types (%)

Arch form	Harmonic / contribution	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}
Square	contribution	25.07	35.46	25.40	3.68	3.76	3.71	2.92
(13 cases)	cumulative contribution	25.07	60.53	85.93	89.61	93.37	97.08	100.00
Round Square	contribution	25.39	35.90	25.68	33.38	3.39	3.36	2.95
(36 cases)	cumulative contribution	25.39	61.29	86.97	90.31	93.69	97.05	100.00
Round	contribution	23.95	33.87	24.46	4.87	4.89	4.87	3.09
(3 cases)	cumulative contribution	23.95	57.82	82.28	87.15	92.04	96.91	100.00
shaped	contribution	26.74	37.82	26.86	2.00	2.06	2.06	2.46
(10 cases)	cumulative contribution	26.74	64.56	91.42	93.42	95.48	97.54	100.00

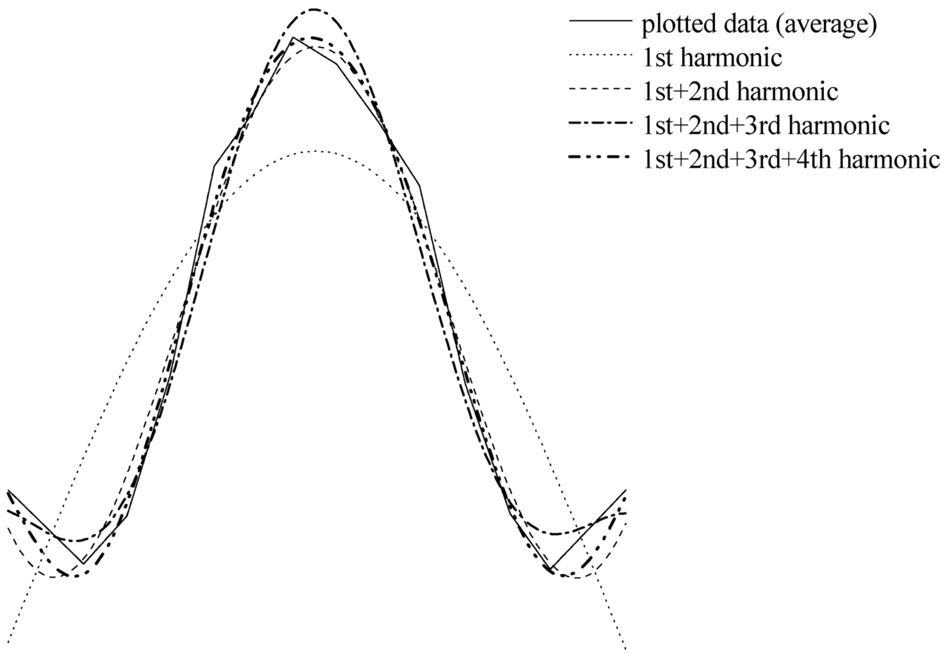

Fig.3：Reproduction of the square dental arch with a Fourier series.
Original data (mean): solid line
1st harmonic: dotted line
1st + 2nd harmonics: broken line
1st + 2nd + 3rd harmonics: 1-dot chained line
1st + 2nd + 3rd + 4th harmonics: 2-dot chained line

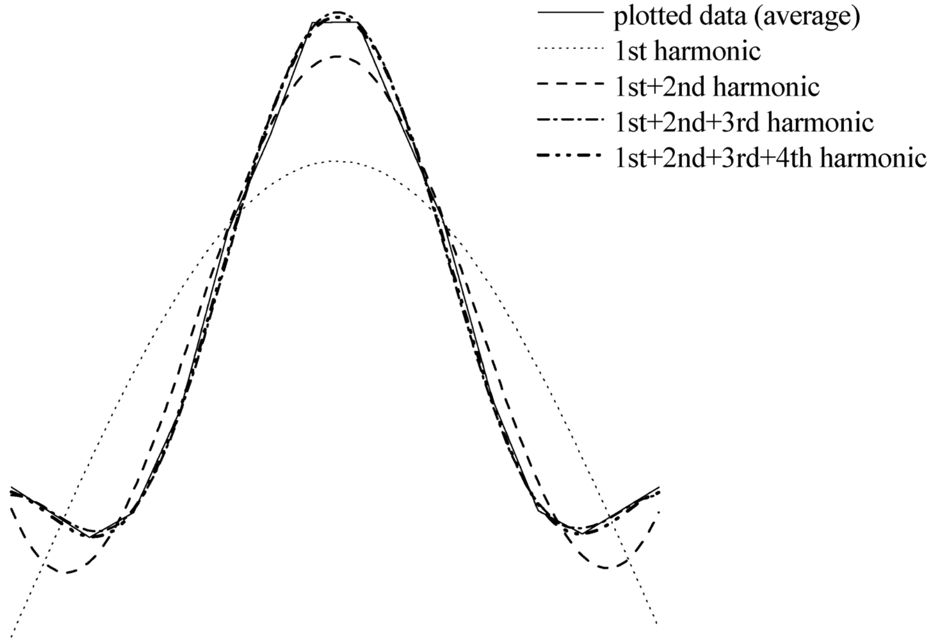

Fig. 4：Reproduction of the round-square dental arch with a Fourier series.

4. Comparison of the constant value and amplitude between different arch types

Significant differences of the constant value ($2a_0$; representing the arch size) and amplitudes of the 1st harmonic (representing the arch forms) were not demonstrated (Table. 2). Scheffe's test was performed for estimation of the differences between amplitudes of the 2nd, 3rd and 4th harmonic that were supposed to represent arch forms. Significant differences for different arch forms of each harmonic were summarized as the following (Table 3-5):

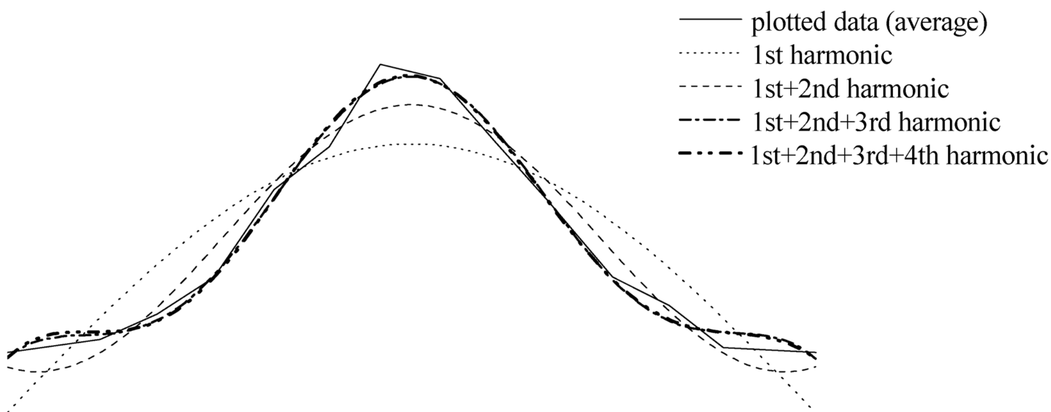

Fig. 5：Reproduction of the round dental arch with a Fourier series.

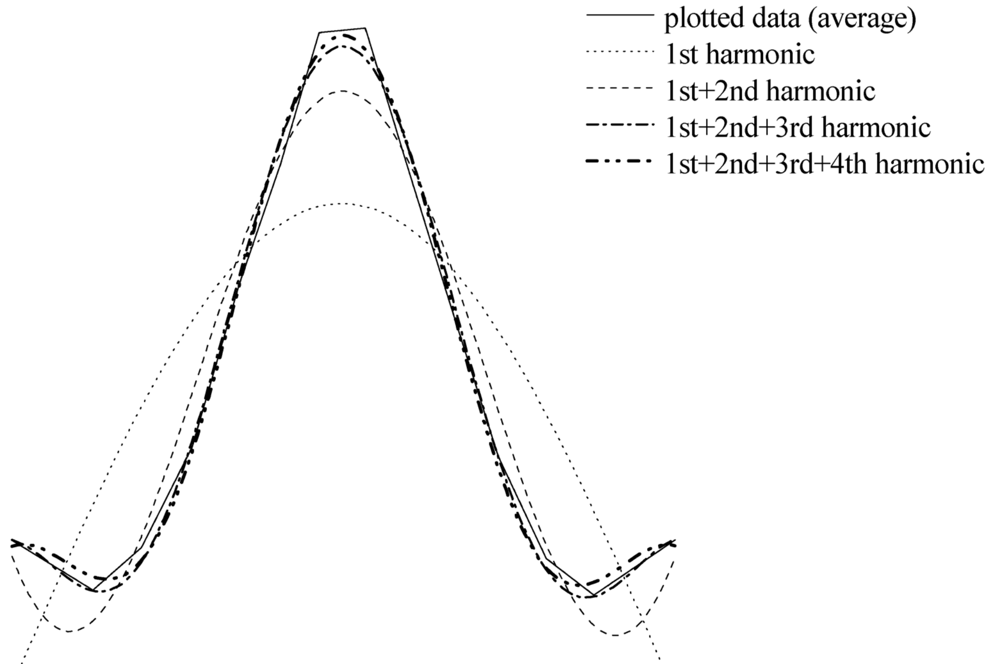

Fig. 6：Reproduction of the round V-shaped dental arch with a Fourier series.

Table 2：Estimation of amplitudes developed from the constant value and harmonics.

	Constant value	Amplitudes			
	value	amplitude	amplitude	amplitude	amplitude
p	0.3126	0.9683	**	**	**

＊＊：p<0.0001

Table 3：Estimation of the 2nd harmonic of different arch types（Scheffe's test）.

	Square	Round Square	Round
Round Square	0.0755		
Round	0.4565	0.9941	
Round V-shaped	**	**	0.3338

＊＊：p<0.0001

Table 4：Estimation of the 3rd harmonic of different arch types（Scheffe's test）.

	Square	Round Square	Round
Round Square	**		
Round	**	0.0966	
Round V-shaped	**	0.2636	0.6680

＊＊：p<0.0001

Table 5：Estimation of the 4th harmonic of different arch types（Scheffe's test）.

	Square	Round Square	Round
Round Square	**		
Round	**	0.4892	
Round V-shaped	0.0656	0.7138	0.2456

＊＊：p<0.0001

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1) The 2nd harmonic: significantly differed between round V-shaped and either square or round-square arch.
2) The 3rd harmonic: significantly differed between square arch and other types of dental arch.
3) The 4th harmonic: significantly differed between square and either round-square or round arch.

5. Comparison of estimated data with the constant value and the amplitude

Relationships between estimated data obtained in our previous study (Nakatsuka *et al.,* 2004) and the constant value and amplitude were represented by correlation coefficients ($p<0.01$; Table. 6). A distinctly positive correlation of the constant value with the anterior-teeth and dental arch width, and a low but positive correlation with the dental length were observed. In the 1st harmonic, a significantly positive correlation between the dental-arch length and anterior-

Table 6：Correlationships of estimated items and the constant value and the amplitudes of different harmonics.

		Amplitudes			
Items	constant value	1st amplitude	2nd amplitude	3rd amplitude	4th amplitude
①	-0.1040	-0.0080	-0.2010	-0.3716**	-0.2333
②	-0.2477	0.1336	-0.7600**	-0.5143**	0.0187
③	-0.2209	0.0882	-0.6207**	-0.4806**	-0.1953
④	-0.2429	0.0548	-0.6131**	-0.4601**	-0.1593
⑤	-0.1696	0.3643**	-0.800**	-0.3428**	0.0940
⑥	0.0324	0.3146**	-0.3574**	0.1026	0.3232**
A-B	0.3731**	0.4868**	-0.5120**	-0.1168	0.0630
A-E	-0.0063	0.1957	-0.5391**	-0.3233**	-0.0063
C_R-C_L	0.7346**	0.4103**	0.3186**	0.4770**	0.0896
$M2_R$-$M2_L$	0.7135**	-0.0228	0.6964**	0.4162**	-0.0716

①$\angle R+\angle L$，②$(A-B)/(C_R-C_L)$，③$180^\circ-\angle(C_R-A-C_L)$，④$(A-E)/(C_R-C_L)$，
⑤$(A-B)/(M2_R-M2_L)$，⑥$(r\theta 5-r\theta 4)_R+(r\theta 5-r\theta 4)_L$

teeth width was found. The 2^{nd} harmonic was highly and positively correlated with the arch width, but was highly and negatively correlated with the anterior-teeth curvature and the arch length-to-width ratio. The 3^{rd} harmonic was positively correlated with the anterior-teeth width and the dental arch width, but was significantly negatively correlated with the anterior-teeth curvature. The 4^{th} harmonic showed significantly low-positive correlation with the difference of radius vectors at the transitional region between the anterior and posterior teeth.

Discussion

1. The contribution and cumulative contribution of maxillary dental arches

The maxillary dental arch forms could be significantly reproduced by 1^{st} to 4^{th} harmonics. The high contributions of 1^{st} to 3^{rd} harmonics indicated that they greatly affected the arch forms. Among them, the 2^{nd} harmonic having highest contribution suggested that 2^{nd}-harmonic curves were the most essential factor affecting the reproducibility of maxillary arch forms. Some previous studies concerning contributions for both deciduous and permanent dentitions have stressed that 1^{st} to 3^{rd} harmonics were essentials representing characters (traits) of dental arch forms, and stated that 1^{st} to 5^{th} harmonics inclusively reproduced the original dental arch forms (Sekimoto *et al.,* 1983; Sekikawa, 1986; Tsuji *et al,* 1986; Sekimoto *et al.,* 1988; Kasai *et al.,* 1997). Some studies have further pointed out that 1^{st} and 2^{nd} harmonics acceptably reproduce the arch forms (Sekikawa, 1986; Kasai *et al.,* 1997). Because of the differences of methods to establish reference points and lines and arch-form comparison, the present results do not directly fit the previous studies, but obtained a common result that maxillary arch forms had reproducibility with lower harmonics. We also noticed that 1^{st} to 3^{rd} harmonics were essential factors, which represented traits for each arch form.

2. The average form for each dental arch form

We superimposed and compared average form with original datum (mean) of each arch form, and found that the 1^{st}+2^{nd}-harmonic curve showed a higher reproducibility than only using the 1^{st}-harmonic curve. Comparing estimated data with the constant value and amplitudes, the present results indicated that the 2^{nd} harmonic was high-and-positively correlated with the arch width, but high-and-negatively correlated with the anterior-teeth curvature, and the arch length-to-width ratio; the relationships were also reflected in the 1^{st}+2^{nd}-harmonic curve of each arch form. We observed that the 1^{st}+2^{nd}+3^{rd}-harmonic curves nearly fitted the original data, and

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indicated 1st, 2nd and 3rd harmonics essentially represented the arch form characters. Furthermore, the 1st+2nd+3rd+4th-harmonic curves showed an increase in the anterior-teeth conformity with original data, as well as reproduced each maxillary dental arch type nearly completely.

3. Superimposition of different average arch forms

The 1st+2nd+3rd+4th-harmonic curves for average arch forms were further superimposed for elucidating arch form traits. We observed that the square arch had a large arch width but a small anterior-teeth curvature than other types; the round-V arch is contrary to that. The round arch was characterised with a gentle transition between anterior- and posterior-teeth curves having small ($r\theta 5$-$r\theta 4$) value in the estimated item ⑥ (Table 6). The results indicated that round-square arch was the average (mostly common) maxillary dental arch.

4. Comparison of the constant value and the amplitude of different arch forms

Some previous studies have pointed out that the constant value in the formula of a Fourier series represented the dental arch size, while the amplitude of a synthetic wave represented traits for the arch forms (Sekimoto, 1983; Sekikawa, 1986; Tsuji *et al.,* 1986; Sekimoto *et al.,* 1988; Kasai *et al.,* 1997). As what have been described in the studies, curves for the 1st-harmonic waves were parabolic shapes in all arch types. Hence the 1st harmonic did not represent characters of different arch types, but indicated that the maxillary dentitions were arranged in a specific arch form. Furthermore, we found that significant differences of the 2nd harmonic between the round V-shaped arch and the square and round-square arches, of the 3rd harmonic between the square arch and other arches, and of the 4th harmonic between the square and the round-square and round arches. The four harmonics showed significant differences between arch types, and therefore considered to be factors affecting the arch form traits.

5. Comparison of estimated data with the constant value and the amplitude

The comparison was performed to study the details of morphological traits, and observed that the constant value had high-and-positive correlation with width of the anterior-teeth segment and dental arch, hence the width was possibly an essential factor affecting the dental arch size. We noticed that the amplitude of the 1st harmonic was in particular positively correlated with the arch length and width of the anterior-teeth segment. The results elucidated that 1st-harmonic curves were morphologically similar in all arch types. However, no significant differences between amplitudes of different arch types were observed, therefore low correlation coefficients in the estimated data indicated that the 1st-harmonic amplitude do not contribute to the morphological traits of any arch type. The 2nd-harmonic amplitude was high-and-positively correlated with the dental arch width, but was high-and-negatively correlated with the anterior-teeth curvature and the arch length and width. The results suggested that the 1st+2nd-harmonic curves in particular fitted with the molar segment, thus tent to represent the morphological traits for different arch forms. The 2nd-harmonic amplitude also reflected the dentition length-to-width ratio, therefore essentially determined the gross appearances of dental arches. Furthermore, we observed that the 2nd-harmonic amplitude had the most-highly contributions in all arch types (Figs. 1 & 2). A previous study has

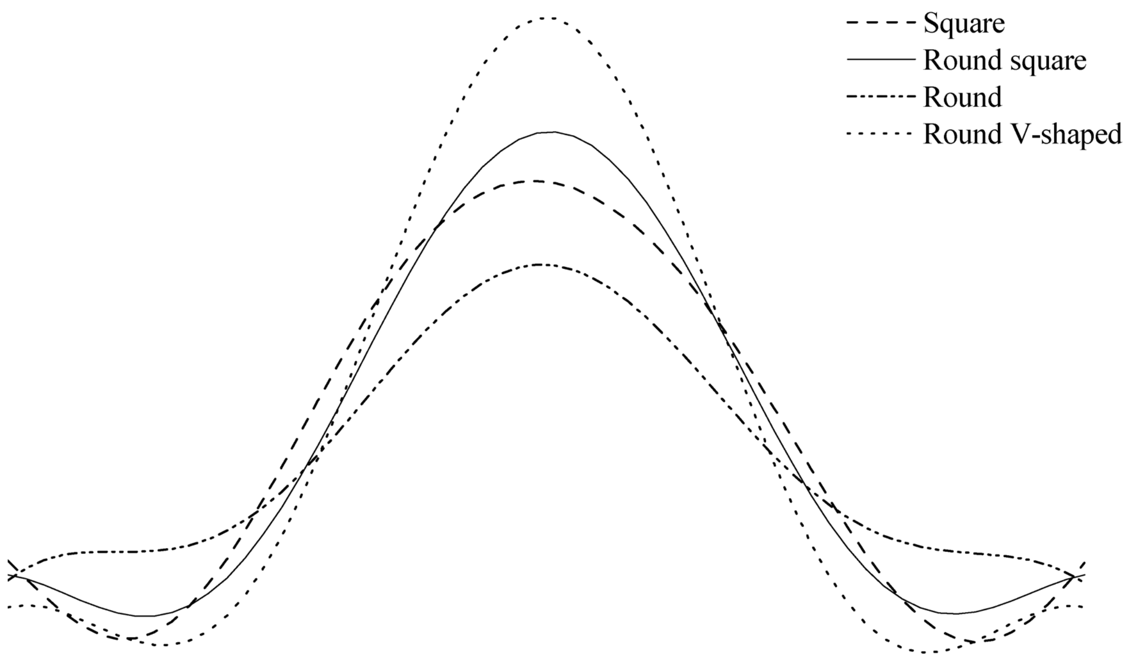

Fig.7：Superimposition of average dental arch forms.
square arch: broken line
round-square arch: solid line
round arch: 2-dot chained line
round V-shaped arch: dotted line

observed that the arch width-to-length ratio was the most prominent factor affecting the morphological variations of maxillary dental arches (Sekikawa, 1986). The present study elucidated that 1^{st}-harmonic curves showed distinct morphological differences with $1^{st}+2^{nd}$-harmonic curves, but did not with the $1^{st}+2^{nd}+3^{rd}$- and $1^{st}+2^{nd}+3^{rd}+4^{th}$-harmonic curves. The results suggested that the dental arch length-to-width ratio significantly determined the arch forms. The 3^{rd}-harmonic amplitude was positively correlated with the width of the anterior-teeth segment and dental arch, but negatively correlated with the anterior-teeth curvature. The $1^{st}+2^{nd}+3^{rd}$-harmonic curves were characterised by an increase in adaptation of the anterior-teeth segment than the $1^{st}+2^{nd}$-harmonic curves, hence the 3^{rd}-harmonic amplitude was estimated to represent the degree of anterior-teeth curvatures. Additionally, we noticed that the 4^{th}-harmonic amplitude was positively correlated with the difference of radius vectors. We concluded that the $1^{st}+2^{nd}+3^{rd}$-harmonic curves took more consideration on the reproducibility of the anterior-teeth segment than the 1st-, $1^{st}+2^{nd}$- and $1^{st}+2^{nd}+3^{rd}$-harmonic curves, thus could finely tune the morphology of transitional areas (roundness affecting the curvilinear contour of dental arches aft canines) and the anterior-teeth segment. A study in the literatures analyzed 1^{st} and 2^{nd} harmonics and stated that the constant value, 1^{st}-harmonic and 2^{nd}-harmonic amplitudes represented the dentition size, arch length-to-width ratio and the squarishness of different dental arches (Sekikawa, 1986). In the present study, we analysed several items, designated 1^{st} and 2^{nd} harmonics of a Fourier series different with other previous studies, and yet obtained a same opinion that both 1^{st} and 2^{nd} harmonics are factors determinative to maxillary dental arch forms.

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Evaluation of Dental Arch Characters by Analysis of a Fourier Series Developed from Mathematically Estimated Maxillary Dentitions

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Our previous principal component analysis conducting on reference points, lines and angles, and a vector-developed polar coordinate system has elucidated that the components of eigenvectors had positive relationships in the curvature of anterior teeth segment, between the protrusion of canines and degree of arch roundness, and in the length-to-width ratio of 62 maxillary dentitions, which were preliminarily classified with reference to the conventional Thompson's morphological descriptions for dental arch forms. In the present study on morphological characters of the maxillary dentitions, we conducted a Fourier analysis on the previously obtained data. We observed that the amplitude of 2nd, 3rd and 4th Fourier harmonics were closely correlated with the length-to-width ratio, curvature of the anterior teeth segment, and the curvilinear contour of maxillary dental arches. In addition, the relationships between previously estimated data and the constant value and the amplitude of the Fourier series were examined by analysis of correlation coefficients ($p<0.01$). The results of the present study suggest that the morphology of maxillary dentitions consists of three essentials –the length-to-width ratio, the curvature of anterior teeth, and the curvilinear contour of dental arches–.

Key Words : Fourier analysis, Maxillary dental arch.

The Taiwan J Oral Med Sci 2008; 24:86-104
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Submitted:March, 2, 2008
Accepted: Apil, 20, 2008